COST ALLOCATION

N. Okada
Disaster Prevention Research Institute, Kyoto University, Japan

K. Tanimoto
Department of Social Systems Engineering, Tottori University, Japan

H. Sakakibara
Department of Civil Engineering, Yamaguchi University, Japan

Keywords: Cooperative game theory, non-cooperative game theory, environmental management, resources allocation, water resources development, international cooperation, core, nucleolus, Shapley value, coalition formation

Contents

Summary

In the case of environmental management or resource management, many stakeholders are involved. By working together, they can often achieve their goals such as reducing

environmental damage and efficiently using the available resources. For example, they can construct a facility at a lower cost by means of establishing a joint project. Although the economic merit of joint work exists among the participants, a conflict among stakeholders may arise. The potential conflict may arise as a result of total cost allocation given that all the participants are assumed to work together. To resolve this conflict, fair cost allocation is necessary. This conflict is called the cost allocation problem and can be modeled as a cooperative game where stakeholders (players) necessarily form a grand coalition. This article shows how to allocate the joint costs by means of cooperative game theory in the context of water resources development and environmental burden allocation such as acid rain and water pollution. We also refer to the conventional cost allocation method used in actual water resources development projects and show the relationship between game theory based methods and conventional techniques. In non-cooperative game theory, players may not form a grand coalition in a non-cooperative game because players' choices are independent and are not constrained. For this case, an inappropriate allocation method may prevent players from participating in a coalition. Whatever the case, coalition formation under cost allocation should be considered in practical situations. We show some stability concepts regarding the formation of a coalition and illustrate these concepts by use of numerical examples in the context of environmental management.

1. Introduction

Let us imagine that there is a joint enterprise (or project) in which multiple stakeholders participate. Assuming you are a potential stakeholder (participant), what would make you motivated to be included in this joint enterprise? Once included, are you sure that your interest would not be in conflict with those of others? By examining these questions, each of you can make a decision as to your involvement in the joint enterprise which might require all the participants to resolve conflicts. In the real world, there are abundant cases of such multiple participants (decision-makers) conflict problems. Amongst them, there is a well-known conventional problem called “cost allocation”.

A central concern here is how to allocate joint costs if we all contemplate joining the entire (grand) enterprise. Of course, each of you can act alone but there is a good reason for each of you to be motivated to join the grand coalition. Most typically, you can assume that it would cost all of you less in total to undertake the grand enterprise, as compared to the case in which each of you act independently, or some of you mutually form a subset of the grand group (coalition). This kind of problem which is called “cost allocation” or “cost sharing” provides an excellent example of the application of game theory explained in this article.

In the field of public infrastructure development, cost allocation has been a significant managerial issue and, therefore, the problem has a long tradition of practically finding fair-division solutions, say by rules of thumb. The disciplines of water resources planning and management have accumulated profound knowledge and valuable experience to deal with cost allocation. For example, multi-purpose reservoir development has heuristically established conventional methods for cost allocation and some of them have been time-tested and proven to be applicable to real-world cases, though applied in a somewhat ad hoc and intuitive manner. As we will see, the twenty-first century is demanding that we readjust our current social schemes to accommodate new trends. The conventional methods of cost allocation will certainly not be an exception and this is why we need to provide a more theoretically sound basis for examining and improving them. Game theory will be introduced for this purpose. To begin with, we will discuss cost allocation by using a multi-purpose reservoir development project.

2. Cooperative Game Theoretic Approach

2.1. Cost Allocation in Resources Management

2.1.1. Description of Cost Allocation in Multi-purpose Reservoir Development

One of the main concerns in resource management is how available resources such as energy and capital should be distributed among uses. A typical case can be found in multi-purpose reservoir development within the field of water resources. Multi-purpose reservoir development is a large- scale project that will produce many benefits such as flood control, municipal water supply, industrial water supply, agricultural water supply, power generation and recreation.

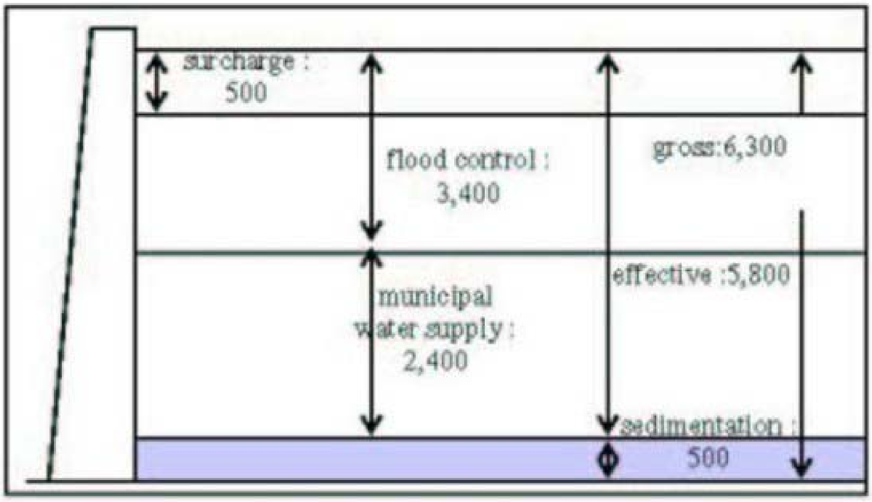

Figure 1. Water volume distribution plan (unit: 1,000 m3)

Water available in a basin is a resource that will be distributed accordingly to the “user” of the reservoir, who is obliged to meet the demand of a given use in what is called resources allocation. The amount of water to be distributed for each use is depicted as a “water volume distribution plan” shown in Figure 1 which depicts the water volume distribution between flood control and municipal water supply use.

The reason why this development can be realized as a joint project is due to the increasing returns in scale in terms of project cost. In particular, the more users participate in the project, the less the average cost becomes. This property comes from the geological condition at the construction site for the reservoir. Since a reservoir is usually constructed in a steep-walled V-shaped valley, additional capacity of the reservoir increases with the height of the dam, which represents the project cost.

Therefore, a reservoir can be developed at a lower cost by having users working together rather than independently. Hence, each user has an incentive to participate in the joint project and multi-purpose reservoir development is assumed to be a joint project consisting of all users that may participate.

Once resources allocation is determined, costs for the projects by partial users can be calculated as well as joint project costs by all users. The former is the alternative cost when a subset of users develops the reservoir by themselves. Joint project cost is allocated to each user based on the alternative costs. In cost allocation, each user wants to minimize his or her allocated cost. Namely, a potential conflict arises when joint project costs must be allocated amongst users. The cost allocation should satisfy the condition for fairness among all users as well as economic efficiency. In the United States, SCRB (Separable Cost Remaining Benefits method) has been in practice as a conventional cost allocation method which was originally proposed by the Tennessee Valley Authority. This method is also used in Japan, although its formulation was adjusted as necessary and it was renamed as the Separable Cost Alternative Benefits method.

2.1.2. Formulation of Conventional Cost Allocation Methods

As conventional cost allocation methods for studying multi-purpose reservoir systems, SCRB and ENSC (Egalitarian Non-Separable Cost method) are well-known.

SCRB (Separable Cost Remaining Benefits method): In this technique, separable cost is first allocated to each user i. Separable cost for user i (SCi) represents the incremental cost when she participates in the project as the last participant. Non-separable cost (NSC) is a reminder of the joint project cost after allocating the separable cost to all users. Then NSC is allocated in proportion to user i's remaining benefits (μi) as:

$$\mu i = Ci - SCi \qquad (1)$$

where, Ci is the cost if user i developed the reservoir only by herself without participating in the joint project. Thus, the total charge for user i is shown below, where N is a set of users.

$$x_i = SC_i + \frac{\mu_i}{\sum_{j \in N} \mu_j} NSC \qquad (2)$$

ENSC (Egalitarian Non-Separable Cost method): As in the case for SCRB, separable cost is first allocated to each user. However, NSC is allocated equally to all users. Thus, the total charge to user i is:

$$x_i = SC_i + \frac{1}{n} NSC \qquad (3)$$

where n is the number of users.

It is a fact that ENSC was not chosen as a cost allocation method by the TVA (TennesseeValley Authority) in the USA, in spite of being recognized as a promising method. The reason for this may be that the equal allocation of NSC does not accurately represent the differences of economical merit by users for participating in the joint project. For the case where the difference is negligible, however, the ENSC is considered to be an effective cost allocation method.

The conventional cost allocation methods are recognized as practical and time-tested allocation techniques. On the other hand, however, there are criticisms that the allocation by a conventional cost allocation method is ad hoc and lacks a proper theoretical background.

2.2. Cooperative Game Model

2.2.1. Concepts in Cooperative Game Theory

Cost allocation in multi-purpose reservoir development can be interpreted as the game situation where each user is a player and each player acts to minimize her allocated cost; she forms a coalition with other players. If some conditions hold, the grand coalition consisting of all players is the most efficient. These and related concepts can be classified as follows:

· player, i ($i \in N$): user participating in the project.

· set of players, N (= {1,2,..,n}) : set of users participating in the project.
· coalition, S ($S \subset N$) : subset of users in a joint project.

· grand coalition, N : coalition consisting of all users participating in a joint project.
· singleton coalition, {i}: coalition consisting of one user only taking part in a distinct project by herself.
· cost function, C(•) : C(S) is an alternative cost for the coalition S and C(N) is the total costs for the joint project consisting of all users, and
· allocated cost, xi: cost allocated to user i.

By using these notations, the concepts needed to explain conventional cost allocation methods can be formulated as:

separable cost for player i: $C(N) - C(N \setminus \{i\})$

non-separable cost: $C(N) - \sum_{j \in N} SC_j$

2.2.2. Conditions for forming the grand coalition

The most important assumption on which cooperative game theory often relies is the formation of the grand coalition. This assumption is satisfied if the following concepts hold.

Sub-additivity: The condition where it is less costly to form a coalition can be explained using a cost function. This condition is called sub-additivity. This condition shows that the larger a coalition is, the more efficient it is. The game is called a sub-additive game when

$$C(S) + C(T) \geq C(S \cup T), \quad (S \cap T = \emptyset;\ S, T \subset N) \tag{4}$$

where, Ø is the empty set and S and T are any two disjoint coalitions. If the game is sub-additive, the grand coalition is the most efficient. Sub-additivity is the necessary condition required for forming the grand coalition.

Core: The conditions which motivate all players or coalitions to join the grand coalition in terms of allocated costs are represented by individual rationality, group rationality and grand rationality. The set of allocations which satisfies these conditions is called the core. Under the assumption that the game is sub-additive, the three conditions are,

$$x_i \leq C(i) \qquad (\forall i \in N) \tag{5}$$

$$\sum_{i \in S} x_i \leq C(S) \qquad (\forall S \subset N) \tag{6}$$

$$\sum_{i \in N} x_i = C(N) \tag{7}$$

It is noted that C({i}) is represented as C(i) for simplicity. Individual and group rationality means that any allocation vector X = (x1,x2,...,xn) in core attains cost saving in comparison with the case in which the coalition or player does not participate in the grand coalition N. Grand rationality reflects economic efficiency and hence the sum of costs separately allocated to each player equals the total cost for the grand coalition. This concept is taken into account in cost allocation when the most efficient coalition, the grand coalition, is formed.

A sub-additivity game implies that each player has an incentive to form the grand coalition and the game with a non-empty core means the existence of fair cost allocation if the players form the grand coalition. However, we still have a problem in determining the unique fair cost allocation. In cooperative game theory, a range of fair cost allocation concepts have been proposed.

Here we have to make sure that all games are not sub-additive. If sub-additivity does not hold, then which coalition should be formed is also of concern for the players. In this case, we have to explore which coalition would be stable under the allocated cost for all coalitions. This extended cost allocation problem can be approached not by a cooperative game theory but by non-cooperative game theory and is discussed in Section 3.5.

2.2.3. Formulation of Cooperative Game Theory Based Methods

Unique solutions derived from core are defined below:
Nucleolus: One of the unique solutions found by reducing or expanding the core is the nucleolus. The nucleolus minimizes the maximum excess of any coalition S lexicographically. The excess of coalition S for allocation vector X is:

$$e(X:S) = \sum_{i \in S} x_i - C(S) \tag{8}$$

The nucleolus is given as X by the formulation:

$$\max_{S \subset N} e(X : S) \to \min$$

such that $\sum_{i \in N} x_i = C(N)$ (9)

The nucleolus is calculated as a solution with fairness by minimizing the maximum excess lexicographical. It can be found by solving at most n – 1 linear programs.
Weak-nucleolus: An alternative approach based on the core can be formulated. In place of e (X : S) the weak-nucleolus defines the average excess ê (X : S) as :

$$\hat{e}(X : S) = e(X : S) / |S| \quad (10)$$

where |S| is the number of players in coalition S.
Propensity to disrupt: In place of e (X : S), propensity to disrupt defines the d (X : S) as:

$$d(X : S) = \frac{-e(X : N \setminus S)}{-e(X : S)} \quad (11)$$

where d (X : S) is a ratio of coalition S's excess and $N \setminus S$'s excess. As a necessary condition for the propensity to disrupt to exist, group rationality must hold:

$$\sum_{i \in S} x_i \le C(S) \quad (12)$$

Note that these nucleoli and their derivations always satisfy the core if it exists. There are other non-core-based approaches.

Shapley value: The Shapley value gives the allocation by averaging the marginal cost for all sequences of coalition formations leading to the grand coalition such that:

$$x_i = \sum_{S \subseteq N, i \in S} \frac{(n-s)!(s-1)!}{n!} [C(S) - C(S \setminus \{i\})] \quad (13)$$

where s is the number of players in coalition S (s = |S|). The Shapley value satisfies axioms such as symmetry, efficiency and law of aggregation. Because the Shapley value is not based on the core, the solution may fail to be in the core even if the core is not empty.

NSCG (Non-Separable Cost Gap method): Other game theoretic approaches which are not based on core are NSCGs. To show the formulation of NSCG, the cost gap function is defined as:

$$g(S) = C(S) - \sum_{i \in S} SC_i \quad (14)$$

If S = N, g(S) then g(N) is equal to NSC. Thus, g(S) can be interpreted as non-separable cost for coalition S. In addition, a concession amount for player i is defined as:

$$\lambda_i = \min_{i \in S} g(S) \tag{15}$$

The g(S) can be interpreted to be the risk for player i (i $\in$ S) because if separable costs have been allocated to all players, player i may have to be allocated the remaining cost, g(S). Thus, the concession amount of player i means the minimum risk for player i.

NSCG argues that SCi should be allocated to player i and NSC is allocated in proportion to a concession amount. Thus, total charge to player i is:

$$x_i = SC_i + \frac{\lambda_i}{\sum_{j \in N} \lambda_j} NSC \tag{16}$$

-
-
-

Bibliography

Carraro, C., and D., Siniscalco ed. (1996): *New Directions in the Economic Theory of the Environment*, Cambridge University Press. [The articles in this book classify coalition formation games into two types, "exclusive membership games" and "open membership games." They also describe the stability of coalition structures.]

Dutta, B. and S. Mutuswami (1997): Stable Networks, *Journal of Economic Theory*, **76**, 322-344. [The Link Formation Game is defined in this paper. Also discussed is the relationship between the stability of graphs and efficiency.]

Federal Inter-Agency River-Basin Committee (1950): Proposed Practices for Economic Analysis of River Basin Projects, Technical Report, Washington D.C. [This is a representative report which investigates cost allocation methods in the multi-purpose reservoir development.]

Myerson, R. B. (1991): *Game Theory - Analysis of Conflict*, Harvard University Press. [The relationship between cooperation structure and the Shapley (or Myerson) value is discussed.]

Okada, N. and K. Tanimoto (1996): Interpreting and Extending Conventional Cost Allocation Methods for Multipurpose Reservoir Developments by Use of Cooperative Game Theory, Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, 2698-2703. [This paper shows the connections between the cooperative game theory cost allocation methods and the conventional cost allocation techniques.]

Okada, N. and Y. Mikami (1992): A Game-Theoretic Approach to Acid Rain Abatement: Conflict Analysis of Environmental Load Allocation, *Water Resources Bulletin*, 28, 155-162. [This paper studies environmental load allocation by use of cooperative game theory.]

Okada, N., K. Takano, H. Tatano, and H. Sakakibara (1998): Efficiency and Stability of Self-Organizing Cooperation Networks: Roles of a Cost Allocation Scheme for Regional Joint Water Distribution Systems, Proceedings of the IEEE International Conference on Systems, Man and Cybernetics.[The relationship between cost allocation rules and coalition formation is discussed.]

Roth. A. ed. (1988): *The Shapley value: Essays in Honor of L.S. Shapley*, Cambridge University Press. [This book introduces *Shapley value* and *Myerson value*.]

Young, H.P. (1994): Cost Allocation, in: R.J. Myerson and S. Hart, ed., *Handbook of Game Theory, Volume 2*, 1193-1235, North-Holland. [This paper presents solution concepts such as the core and nucleolus.]

Young, H.P. ed. (1985): *Cost Allocation: Methods, Principles, Applications*. Amsterdam: North-Holland. [This book presents cost allocation methods and their applications.]

Biographical Sketches

Norio Okada received his B.Eng. degree from the Department of Sanitary Engineering, Kyoto University in 1970, Ms.Eng from Kyoto University in 1972, and Dr. Eng. from Kyoto University in 1977. He received an Honoris Causa honourary doctorate in Engineering from the University of Waterloo, Canada, in 1996, and Honorary Doctor of Eng., from Chong Quing Communication University, China, in 1998. He is currently Professor of Disaster Risk Management at the Disaster Prevention Research Institute (DPRI), Kyoto University. His major research interests are game theory, disaster and environmental risk management, and regional planning. Dr. Okada is a member of the Japan Society of Civil Engineers (JSCE), Society of Risk Analysis, Japan Society of Hydrology and Water Resources and many others organizations. In 1995, he received JSCE Research Awards for his application of game theory to cost allocation in water resources development.

Keishi Tanimoto received his B.Eng., Master's and Ph.D. degrees in Civil Engineering from Kyoto University, Japan. Dr. Tanimoto is currently an Associate Professor in the Department of Social Systems Engineering, Tottori University, Tottori, Japan. Dr. Tanimoto has actively carried out research and consulting activities in the areas of social system engineering, conflict analysis, and decision making, particularly in interactive decision making and decision support systems. Dr. Tanimoto is a member of the Japan Society of Civil Engineers (JSCE), and Operations Research Society of Japan. In 1995, he received JSCE Research Awards for his application of game theory to cost allocation in water resources development.

Hiroyuki Sakakibara received his B.S. degree in Civil Engineering, M.S. degree in Applied System Science, and his Ph.D. degree in Civil Engineering from Kyoto University, Kyoto, Japan, in 1995, 1997, and 2001, respectively. He is an Assistant Professor in the Department of Civil Engineering, Yamaguchi University, Ube, Yamaguchi, Japan. His major research interests are the application of game theory to the management of infrastructure projects, evaluation of urban structures, and decision making in disaster risk management. Dr. Sakakibara is a member of the Japan Society of Civil Engineers (JSCE), Society of Risk Analysis, Institute of Electric and Electronic Engineers (IEEE) and The Japan Society of Hydrology and Water Resources.